Pearson Edexcel Level 3

GCE Mathematics

Advanced

Paper 1: Pure Mathematics — PMT Mock 1

Time: 2 hours	Paper Reference(s)
	9MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. a. Find the first four terms, in ascending powers of x, of the binomial expansion of

$$(\tfrac{1}{9} - 2x)^{\frac{1}{2}}$$

giving each coefficient in its simplest form.

(4)

b. Explain how you could use $x = \frac{1}{36}$ in the expansion to find an approximation for $\sqrt{2}$.

There is no need to carry out the calculation.

(2)

(Total for Question 1 is 6 marks)

2. The curves C_1 and C_2 have equations

$$C_1: \quad y = 2^{3x+2}$$

$$C_2: \quad y = 4^{-x}$$

Show that the x-coordinate of the point where C_1 and C_2 intersect is $\frac{-2}{5}$.

(3)

(Total for Question 2 is 3 marks)

3. Relative to a fixed origin,

- point A has position vector $-2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$
- point B has position vector $-\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$
- point C has position vector $\mathbf{i} + \mathbf{j} + 4\mathbf{k}$
- point D has position vector $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

a. Show that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel and the ratio $\overrightarrow{AB}$: $\overrightarrow{CD}$ in its simplest form.

(4)

b. Hence describe the quadrilateral *ABCD*.

(1)

(Total for Question 3 is 5 marks)

4. Ben starts a new company.

- In year 1 his profits will be £24000.
- In year 11 his profit is predicted to be £64000.

Model ***P*** assumes that his profit will increase by the same amount each year.

a. According to model ***P***, determine Ben's profit in year 5.

(3)

Model ***Q*** assumes that his profit will increase by the same percentage each year.

b. According to model ***Q***, determine Ben's profit in year 5. Give your answer to the nearest £10.

(3)

(Total for Question 4 is 6 marks)

5. The function f is defined by

$$\mathrm{f}: x \to \frac{2x-3}{x-1} \qquad x \in R, \;\; x \neq 1$$

a. Find $\mathrm{f}^{-1}(3)$.

(2)

b. Show that

$$\mathrm{ff}(x) = \frac{x+p}{x-2} \qquad x \in R, \;\; x \neq 2$$

where p is an integer to be found.

(3)

The function g is defined by

$$g: x \to x^2 - 5x \qquad x \in R,\ \ 0 \leq x \leq 6$$

c. Find the range of g.

(3)

d. Explain why the function g does not have an inverse.

(1)

(Total for Question 5 is 9 marks)

6. a. Express $4\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

Give the exact value of R, and give the value of α, in degrees, to 2 decimal places.

(3)

$$T = \frac{8400}{19+(4\sin x - 5\cos x)^2}\ , \quad x > 0$$

b. Use your answer to part *a* to calculate

i. the minimum value of T.

ii. the smallest value of x, $x > 0$, at which this minimum value occurs.

(4)

(Total for Question 6 is 7 marks)

7.

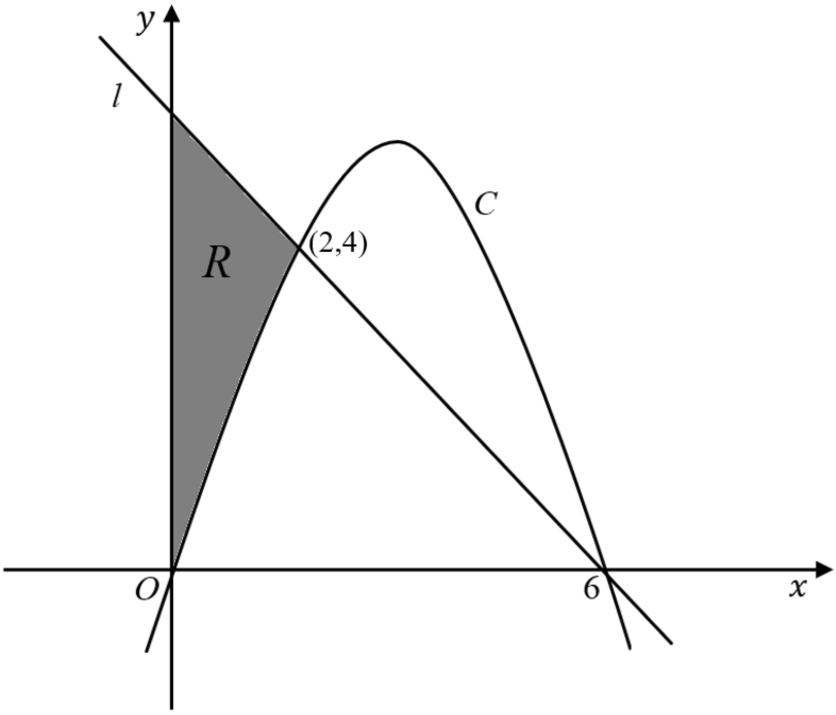

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ and a straight line l.

The curve C meets l at the points (2,4) and (6,0) as shown.

The shaded region R, shown shaded in Figure 1, is bounded by C, l and the y-axis.

Given that $f(x)$ is a quadratic function in x, use inequalities to define region R.

(5)

(Total for Question 7 is 5 marks)

8.

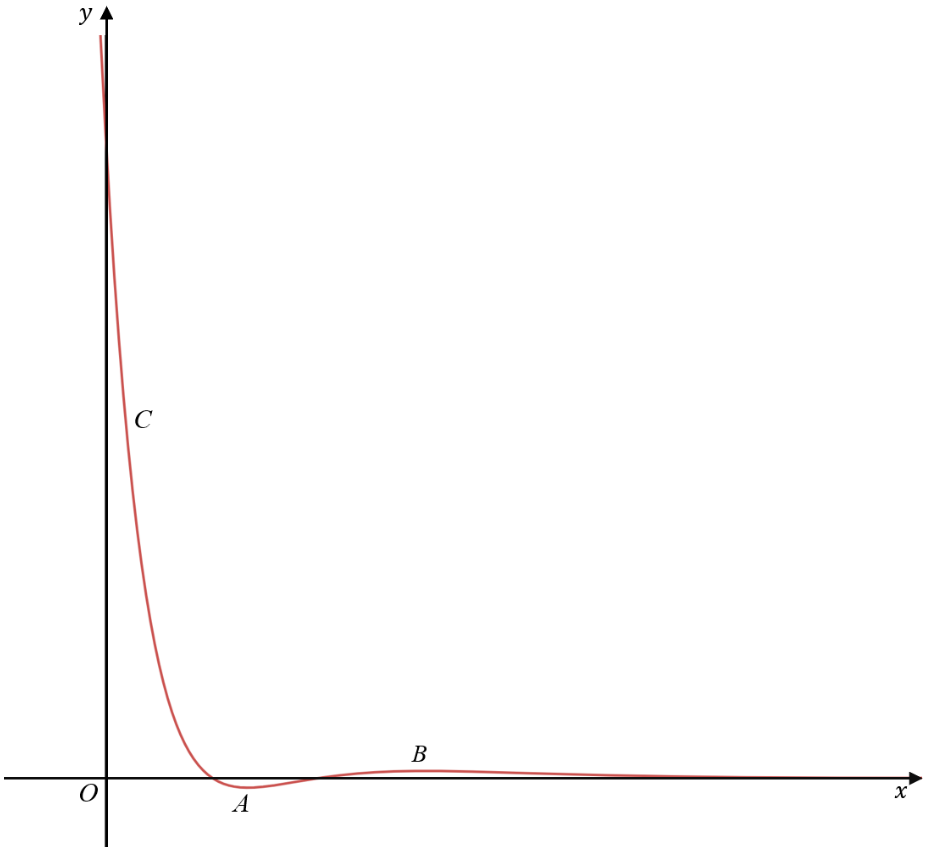

Figure 2

Figure 2 shows a sketch of the curve C with the equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (2x^2 - 9x + 9)e^{-x}, \quad x \in R$$

The curve has a minimum turning point at A and a maximum turning point at B as shown in the figure above.

a. Find the coordinates of the point where C crosses the y–axis.

(1)

b. Show that $f'(x) = -(2x^2 - 13x + 18)e^{-x}$

(3)

c. Hence find the exact coordinates of the turning points of *C*.

(3)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation

$$y = a\mathrm{f}(x) + b\,, \qquad x \geq 0$$

The range of the graph with equation $y = a\mathrm{f}(x) + b$ is $0 \leq y \leq 9e^2 + 1$

Given that a and b are constants.

d. find the value of a and the value of b.

(2)

(Total for Question 8 is 9 marks)

9. a. Use the substitution $t^2 = 2x - 5$ to show that

$$\int \frac{1}{x+3\sqrt{2x-5}} \ \mathrm{d}x = \int \frac{2t}{t^2+6t+5} \ \mathrm{d}t$$

(3)

b. Hence find the exact value of

$$\int_3^{27} \frac{1}{x+3\sqrt{2x-5}} \ \mathrm{d}x$$

(5)

(Total for Question 9 is 8 marks)

10.

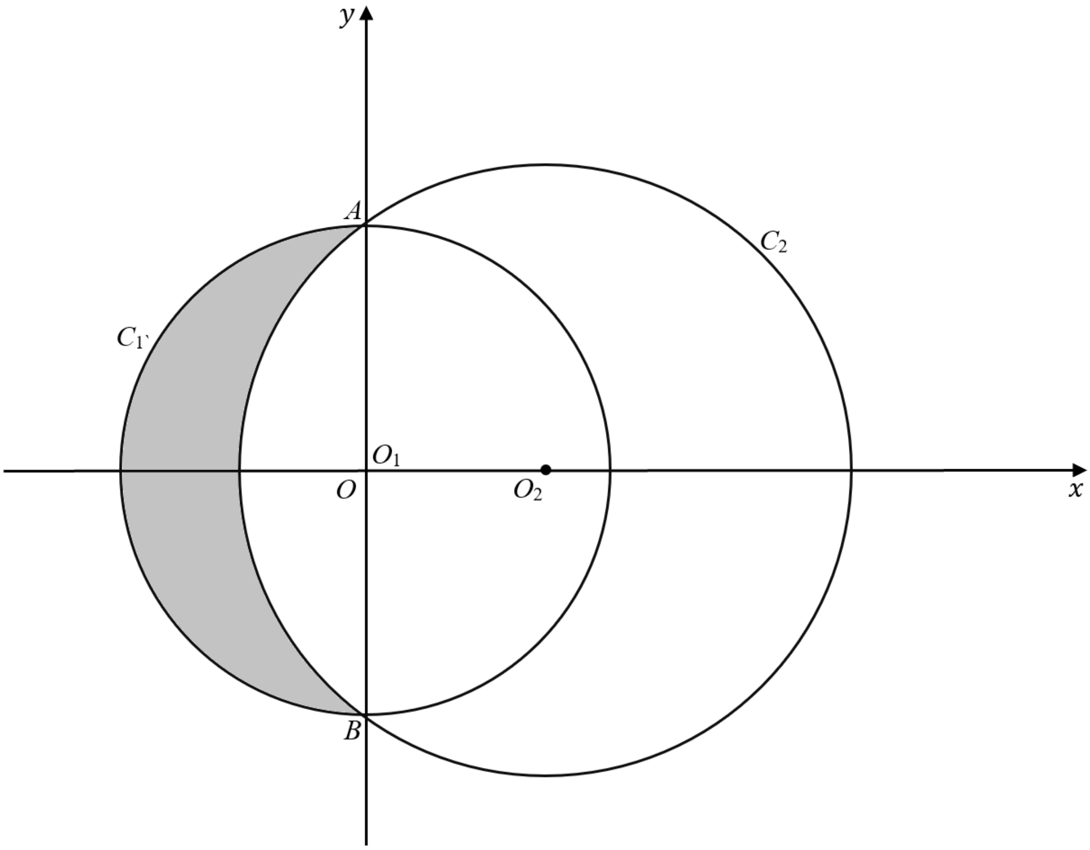

Figure 3

Circle C_1 has equation $x^2 + y^2 = 64$ with centre O_1.

Circle C_2 has equation $(x - 6)^2 + y^2 = 100$ with centre O_2.

The circles meet at points A and B as shown in Figure 3.

a. Show that angle $AO_2B = 1.85$ radians to 3 significant figures.

(3)

b. Find the area of the shaded region, giving your answer correct to 1 decimal place.

(3)

(Total for Question 10 is 6 marks)

11. In a science experiment, a radio active particle, N, decays over time, t, measured in minutes. The rate of decay of a particle is proportional to the number of particles remaining.

Write down a suitable equation for the rate of change of the number of particles, N in terms of t.

(2)

(Total for Question 11 is 2 marks)

12. a. Show that

$$\sec\theta - \cos\theta = \sin\theta\tan\theta \qquad \theta \neq (\pi n)^0 \quad n \in Z$$

(3)

b. Hence, or otherwise, solve for $0 < x \leq \pi$

$$\sec x - \cos x = \sin x \tan(3x - \frac{\pi}{9})$$

(5)

(Total for Question 12 is 8 marks)

13. A sequence $a_1, a_2 a_3, \ldots$ is defined by

$$a_{n+1} = 5 - pa_n \qquad n \geq 1$$

where p is a constant.

Given that

- $a_1 = 4$
- the sequence is a periodic sequence of order 2.

a. Write down an expression for a_2 and a_3.

(2)

b. Find the value of p.

(2)

c. Find $\sum_{r=1}^{21} a_r$

(2)

(Total for Question 13 is 6 marks)

14. A circular stain is growing.

The rate of increase of its radius is inversely proportional to the square of the radius.

At time t seconds the circular stain has radius r cm and area A cm^2.

a. Show that $\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{k}{\sqrt{A}}$.

(4)

Given that

- the initial area of the circular stain is 0.09 cm^2.
- after 10 seconds the area of the circular stain is 0.36 cm^2.

b. Solve the differential equation to find a complete equation linking A and t.

(6)

(Total for Question 14 is 10 marks)

15. The curve C has equation

$$y = \frac{1}{2}x - \frac{1}{4}\sin 2x \qquad 0 < x < \pi$$

a. Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \sin^2 x$

(3)

b. Find the coordinates of the points of inflection of the curve.

(3)

(Total for Question 15 is 6 marks)

16. Use algebra to prove that the product of any two consecutive odd numbers is an odd number.

(4)

(4)

(Total for Question 16 is 4 marks)